INCOME INEQUALITIES: AXIOMS OF INCOME INEQUALITY MEASURES AND PEOPLE'S PERCEPTIONS

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Abstract: *Studies show that people perceive income inequality differently than most popular income inequality measures and axioms (postulated properties of inequality) indicate. This article synthesizes and reviews different results on income inequality perception. It presents basic income inequality axioms and analyses the level of support found in multiple studies. The paper shows that while answers to particular questions seldom perfectly agree with income inequality measures, the general pattern of respondents' answers is strongly correlated with these measures. It also argues that the observed differences can be partially explained by following Amartya Sen's suggestion that income inequality is a multidimensional concept (1973, p. 48).*

Key words: *Income Inequality, Inequality Axioms, Questionnaire Experiments, Transfer Principle, Population Symmetry.*

JEL code: *D63 (Equity, Justice, Inequality, and Other Normative Criteria and Measurement).*

NIERÓWNOŚCI DOCHODOWE: AKSJOMATY MIAR NIERÓWNOŚCI A OPINIE LUDZI

Streszczenie: *Badania pokazują, że ludzie postrzegają nierówność dochodów odmiennie od wskazań najczęściej stosowanych miar nierówności i aksjomatów (postulowanych własności nierówności). Niniejszy artykuł analizuje i porównuje ze sobą wyniki różnych badań dotyczących postrzegania nierówności dochodów. Opisuje podstawowe aksjomaty pomiaru nierówności wraz z poziomem poparcia wyrażonym przez respondentów w badaniach. Artykuł pokazuje, że o ile odpowiedzi na pojedyncze pytania rzadko są idealnie zgodne z miara-*

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mi nierówności, o tyle ogólny wzór odpowiedzi respondentów jest silnie z tymi miarami skorelowany. Argumentuje również, że obserwowane różnice między odpowiedziami badanych a miarami nierówności mogą zostać wyjaśnione przez zastosowanie sugestii Amartyi Sena, że nierówność dochodów jest koncepcją wielowymiarową (1973, str. 48).

Słowa kluczowe: *nierówność dochodów, aksjomaty nierówności, eksperymenty ankietowe, reguła transferów, symetria populacji.*

1. Introduction

Important concepts of justice, inequality and welfare have no universal definition. There is an ongoing discussion among proponents of different concepts. Recently, a series of empirical research was conducted to elicit people's views and attitudes towards those concepts. For instance, there have been numerous studies on whether people accept rules proposed by distributive justice theory (e.g. Lissowski, Tyszka and Okrasa, 1991, Frohlich, Oppenheimer, Eavy, 1992, Elster, 1995, Gaertner, Schokkaert, 2011) and on perception of income inequality (e.g. Harrison, Seidl, 1994, Gaertner, Namazie, 2003, Amiel, Cowell, Gaertner, 2012).

Inquiries into people's perceptions and definitions of inequality were initiated by Frank Cowell (1985) with an informal questionnaire conducted among students and co-workers. He was surprised to discover that even people with a degree in economics perceived income inequality inconsistently with indications of the most popular income inequality measures. This experience motivated Cowell to conduct a series of questionnaire experiments with Yoram Amiel (e.g. Amiel, Cowell, 1992, 1999, 2002 and with Gaertner, 2012) that were continued and modified by other researchers (Ballano, Ruiz-Castillo, 1994, Gaertner, Namazie, 2003, Jancewicz, 2013, 2014).

Studies show that people's evaluations significantly differ from indications of income inequality measures and axioms, i.e. postulated properties of inequality. The magnitudes of the differences (measured as the percentage of answers not in line with income inequality axioms) depend on the sample, exact methodology (for instance: the choice of examples and the precise wording used) and so on, but the general trends are consistent. The unique contribution of this article is that it reviews and synthesizes results obtained in this field of research. Following the notion that respondents' answers should be interpreted holistically, the article finds that while respondents evaluate pairs of income distributions differently than income inequality measures, the general pattern of their responses is strongly correlated with these measures (Jancewicz 2014 and own research conducted in 2014). The paper points

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out that the overall results are in line with Amartya Sen's suggestion that income inequality is a multidimensional concept (1973, p. 48).

The article is organized as follows. The next section presents selected income inequality measures. The sections that follow introduce the basic income inequality axioms and analyze their support in different studies on income inequality perception: the population principle (section 3), scale and translation invariance (section 4), the Pigou-Dalton transfer principle (section 5) and other transfer principles (section 6). Section 7 describes income inequality measures through selected axioms. Section 8 attempts to explain the differences between evaluations of income inequality axioms and people's views.

2. Income inequality concepts and measures

Many different inequality measures exist that represent the multitude of income inequality concepts (Coulter, 1989, Cowell, 2011). Nevertheless, new general concepts of income inequality are formulated rarely, even less frequently than concepts of distributive justice. In both scientific fields, researchers focus on distribution evaluation methods. In the case of distributive justice, these are the rules used to allocate goods; in the case of income inequality, these are the measures considered to be the operational definitions of inequality.

The most popular inequality measures are: the Gini coefficient, the Theil index and quintile share ratio (often written as S80/S20; the sum of incomes of the top 20% divided by the sum of incomes of the bottom 20% of the population). Traditionally, measures are divided into descriptive and normative, depending on whether they use explicit welfare functions or not (Atkinson 1970, Kolm 1969, Sen 1973). This division is not exclusive though, since measures can be reformulated in terms of utility functions, thus presenting descriptive measures as normative ones (e.g. Blackorby and Donaldson, 1978, Gini and Theil – p. 69-70). Another way of classification is through axioms, i.e. proposed income inequality properties, that a measure either satisfies or not – this is an exclusive division.

Notation

The following notation is used in this paper. The examples present incomes divided among people or regions (depending on the example). Each possible division of incomes is called a distribution. Distributions are described by a vector $\mathbf{x} = (x_1, x_2, x_3, ..., x_n$, where numbers $x_1, x_2, ..., x_n$ denote incomes of each person or region.

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Additionally we assume anonymity: it is unimportant who holds the income, only the amount of income matters. Thus, income inequality is independent of the order in which incomes are presented and we can operate on vectors of incomes placed in a non-descending order: $x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n$. It is also assumed that each income is non-negative: $x_i \geq 0$. Mean income is written as $\mu(\mathbf{x}) = \frac{\sum_{i=1}^{n} x_i}{n}$. The set of all possible income distributions is denoted by **D** and the set of all distributions among n income recipients by $\mathbf{D^n}$. The fact that **x** is considered more unequal than **y** is written as $\mathbf{x} >_r \mathbf{y}$ and the fact that **x** and **y** have the same level of income inequality is written as $\mathbf{x} =_r \mathbf{y}$.

The Gini coefficient

The Gini coefficient is one of the oldest income inequality measures (proposed by Corrado Gini in 1914), and also the most frequently used one. Its creation and interpretation is directly linked to the Lorenz curve, which depicts the relationship between cumulative population shares and cumulative income shares. It is defined by a population ordered according to incomes: from the smallest to the largest. Each point says what share of overall income is received by a certain share of population. For simplicity's sake, we discuss only the case of finite populations. We draw the Lorenz curve by marking points (0, 0), (1, 1) and the points corresponding to each member of the population, then we finish by connecting the subsequent points with a straight line (Arnold, 2008).

$$L(x_i)\colon \left({}^{i}/_{n}, \frac{\sum_{j=1}^{i} x_j}{\sum_{j=1}^{n} x_j} \right)$$

The Lorenz curve representing an equal division of income is a straight line from (0,0) to (1,1). The curve depicting maximal inequality consists of two pieces: first one lays flatly as it goes from (0,0) to $\left(\frac{n-1}{n}, 0\right)$ and the second one – showing that all income is received by one person (the 1/n of the population) – goes from $\left(\frac{n-1}{n}, 0\right)$ to (1,1), which is shown in Figure 1.

The value of the Gini coefficient is twice the field size between the line of equality and the Lorenz curve. It equals 0 when everyone has the same income and (1-1/n) in the case of perfect inequality. The traditional form of the Gini coefficient is not the only possible one (many of its forms are presented in the paper by Yitzhaki, Schechtman, 2013). Other interpretations of the Gini coefficient are: a normalized mean of differences of each pair of incomes (formula G2); and a normalized function of incomes weighted by order rank (from smallest to largest incomes. formula G3).

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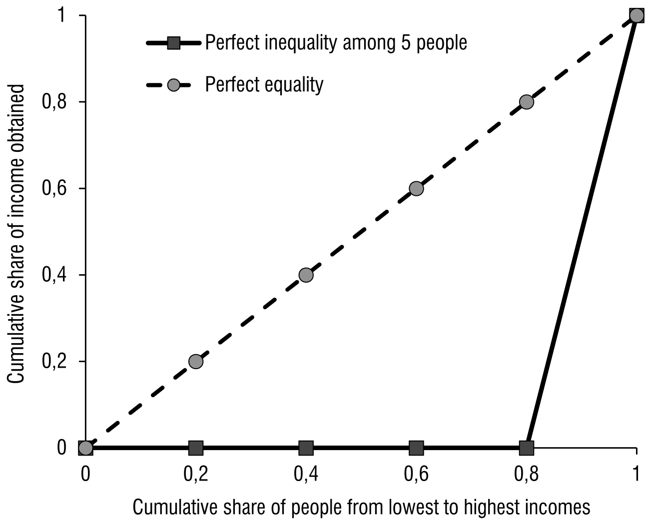
Different formulations of the Gini coefficient expose different properties of this measure.



Example of perfect equality

Person	Income
A	2 000 zł
B	2 000 zł
C	2 000 zł
D	2 000 zł
E	2 000 zł

Example of perfect inequality

Person	Income
A	0 zł
B	0 zł
C	0 zł
D	0 zł
E	10 000 zł

***Figure 1*. The example of Lorenz curve**

G1. $G(\mathbf{x}) = 1 - 2 \cdot \int_0^1 L(\mathbf{x})dx$

The L(**x**) is the Lorenz curve of **x** (here, only the case of a finite population is considered, but this equation also applies when the Lorenz curve is defined for an infinite population)

G2. $G(\mathbf{x}) = \dfrac{1}{2\mu(\mathbf{x}) \cdot n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} | x_i - x_j$

G3. $G(\mathbf{x}) = \dfrac{2}{n^2 \cdot \mu(\mathbf{x})} \sum_{i=1}^{n} (i \cdot x_i) - \dfrac{n+1}{n}$

The Theil index

The Theil index is a modification of entropy (a coefficient related to information theory, shortly reviewed in Maasoumi 1993), adjusted to meet the expectations of basic income inequality axioms (e.g. the population principle). It was proposed by Henri Theil in 1967. Entropy relies on probabilities, while the Theil index substitutes them with income shares of one's income relative to the total income. It is normalized (by subtracting *ln(n)*) to have the value 0 when all incomes are equal, and value

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ln(n) in the case of perfect inequality. The Theil index is hard to interpret (because it uses logarithms), but it allows for the decomposition of inequality by population subgroups[2], which is used to analyze the structure of inequality.

$$T(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{\mu(\mathbf{x})} ln \frac{x_i}{\mu(\mathbf{x})} \mathrm{k}$$

The income quintile share ratio

The income quintile share ratio, denoted as S80/S20, was included in a set of basic European indicators of poverty and social exclusion that require constant monitoring (the Leaken indicators, Eurostat, 2003). S80/S20 is a quotient of sums of incomes for the top and bottom 20% of the population – it is sometimes confused with P80/P20, which is a quotient of incomes of people at the border of those 20%. While the main advantage of S80/S20 is its simplicity, this is also its biggest disadvantage: it ignores the differences within the top and bottom 20% as well as within the middle 60% of population.

$$s80/s20(\mathbf{x}) = \frac{\sum_{\lceil i=0,8 \cdot n \rceil}^{n} x_i}{\sum_{j=1}^{\lfloor 0,2 \cdot n \rfloor} x_j}$$

Summary

Income inequality measures have different properties. Characterizing measures according to their axiomatic properties makes comparing concepts of inequality represented by these measures easier. The most popular income inequality measures have many different axiomatizations – for the Gini coefficient these are for instance: Weymark, 1981, Thon, 1982, Yaari, 1988, Bossert, 1990, Porath, Gilboa, 1994, Aaberge, 2001, Plata-Pérez, Sánchez-Pérez, Sánchez-Sánchez, 2015; and for the Theil index: Bourguignon, 1979, Foster, 1983, Lasso de la Vega, Urrutia, Volij, 2013. However, this article needs to introduce only a few basic axioms instead of axiomatizations.

3. Axioms and their support

In discussions between proponents of different income inequality concepts, a basic set of axioms is considered. Empirical research on income inequality perceptions usually test acceptance for this basic set of axioms, and some of these results are presented in the following sections of this article.

[2] The decomposition axiom postulates that income inequality can be split into inequality coming from within the groups and differences between those groups.

Amiel and Cowell's methodology

Most studies on income inequality perceptions use the methodology proposed by Amiel and Cowell (1992), a brief description of which follows. The samples are non-random and they consist of undergraduate students who had no training in income inequality measurement. The questionnaires are conducted during classes. In Amiel and Cowell's methodology each question concerns one axiom (a proposed property of income inequality) and can have either a numerical or a verbal form. In the first part of the questionnaire, respondents are presented with a series of numerical questions, each one asking them to choose the more unequal of two given distributions. In the second part of the questionnaire, respondents are presented with verbal questions describing a change in an income distribution, and are asked to evaluate how this change would influence inequality. After each verbal question, respondents are encouraged to rethink their answers to the numerical question(s) concerning the same axiom. Studies deviating from this standard are rare – these are either based on different theoretical structures (for instance: Devooght, 2003) or mimicking research on perception of welfare and risk (for instance: Bernasconi, 2002), or significant modifications of Amiel and Cowell's methodology (own research).

The population principle

The population principle (sometimes called population independence) declares that multiplying a society leaves income inequality unchanged. For instance, according to the population principle, inequality of a two-person society: (0, 10000) is the same as inequality of a four-person society: (0, 0, 10000, 10000). The population principle allows for comparing income inequality levels among societies of different sizes. Formally, this axiom can be written as:

$$\forall_{x \in D^n} \forall_{k \in N_+} \quad \mathbf{x} =_r \underbrace{(\mathbf{x}, \mathbf{x}, \mathbf{x}, \mathbf{x}, \ldots, \mathbf{x})}_{k}$$

According to different studies, the idea that multiplying societies doesn't affect inequality is accepted by 40-66% of respondents. Harrison and Siedl (1994) report that only 39,7% of respondents answered that a doubled vector has the same income inequality as the original one. In the first Amiel and Cowell experiment (1992) it was 58%, in a replication of their study on a Polish sample it was 51% (Jancewicz, 2013). When the population principle was stated verbally (again only for doubling the population size) it convinced 66% of Amiel and Cowell's (1992) participants and 60% of Polish participants (Jancewicz, 2013). 45% of the Polish sample agreed with the axiom both in verbal and numerical questions (similar results – 47.9% – were obtained in another Polish sample (Jancewicz, Karpiński, 2013)[3]).

[3] Reformulating the questions and grouping them according to topic instead of type increased the number of answers in line with the population principle to 65,1%. Changes introduced in the modified questionnaire,

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The population principle is the only axiom dealing with populations of different sizes, so most concepts of income inequality accept it. Nevertheless comparing income inequality of a small population to a multimillion-member society is a dubious operation. Around half of the respondents expressed doubt even in the case of population doubling.

Scale and translation invariance

Societies differ in terms of the average income received by their members, thus we need an axiom that describes how to treat those differences when measuring income inequality. There are two rivalling rules:

1. **Scale invariance** states that multiplying incomes by a positive constant leaves income inequality unchanged. For example, distributions such as: (5, 8, 10), (10, 16, 20) and (1000, 1600, 2000) have the same level of income inequality, because they are the result of multiplication of the first distribution by 2 and 200. Scale invariance implies that: proportional changes in real incomes, changes in currency used (e.g. denomination), and changes of currency exchange rates, leave income inequality unchanged.

$$\forall_{c \in R_+} \forall_{x \in D^n} \mathbf{x} =_r c \cdot \mathbf{x}$$

2. **Translation invariance** postulates that adding a constant to all incomes leaves income inequality unchanged. For example, distributions such as: (5, 8, 10), (10, 13, 15) and (1005, 1008, 1010) have the same level of income inequality, because they are the result of adding 5 and 1000 to the first distribution. Advocates of translation invariance argue that proportionate income gains increase inequality and currency exchange rates are important for comparing distributions of countries using different currencies.

$$\forall_{c \in R_+} \forall_{x \in D^n} \mathbf{x} =_r (\mathbf{x} + c \cdot 1^n)$$

Accepting both of these axioms is impractical: it can lead to evaluating distributions such as: (1005, 1008, 1010) and (1000, 1600, 2000) as equally unequal, because they are both modifications of (5, 8, 10). In general, assuming both axioms simultaneously can mean considering many distributions (e.g. all two-people distributions) to be the same from an income inequality perspective.

Most of the popular income inequality measures assume scale invariance, because it solves many problems: proportional changes in real incomes or the currency in which the incomes are calculated have no bearing on the analysis result. Amartya

however, are significant and make the results obtained difficult to compare with other studies that use Amiel and Cowell's classical methodology (1992, 1999).

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Sen, in his book "On Economic Inequality" refuses to accept scale or translation invariance (1973, p. 71). However, he advises using a measure that fulfils scale invariance and then personally evaluating the impact of differences in mean incomes on the results obtained.

Surprisingly, respondents are divided between scale and translation invariance. Acceptance rates measured as percentages of answers in line with one of the axioms in a single numerical question (income distribution examples) are: 37-51% in case of scale invariance and 18-55% in case of translation invariance (Table 2). Similarly, in verbal questions 34-47% support scale invariance and 30-65% support translation invariance (Table 3).

The huge differences in support for translation invariance are due to its large support in Polish samples (Jancewicz, 2013, Jancewicz, Karpiński, 2013) and small support in samples from other countries. All of the samples were non-random, so it is difficult to make meaningful comparisons; it is worth noting though that the Polish respondents often take part in a statistics course where variance (which is translation invariant) has been introduced, so this could have influenced their opinions.

Amiel and Cowell analyzed answers to questions concerning scale and translation invariance jointly, hoping that respondents would choose one axiom over the other. However many respondents agreed with both axioms: 9-24% in numerical questions and 8.5-29% in verbal ones. Amiel and Cowell considered an answer agreeing with both axioms as "illogical under the assumptions normally made concerning inequality measures" (1992, p. 12) and suspected them to be the result of respondents choosing the answer "the same" as a "safe" option in the absence of "I don't know". Amiel and Cowell considered proponents of translation invariance (marked in Table 1 with "*") as those who chose that:

1. Doubling the incomes increases inequality.
2. Adding a positive constant leaves inequality unchanged.

and proponents of scale invariance (marked in Table 1 with "**") as those who chose:

1. Doubling the incomes leaves inequality unchanged.
2. Adding a positive constant decreases inequality

Interpreting answers about both axioms jointly in multiple researches led to classifying 13-27% of answers to numerical and 13-30% to verbal questions as in line with translation invariance (Tables 2 and 3). Similarly, 17-26% of answers to numerical and 17-24% to verbal questions were classified as in line with scale invariance (again Tables 2 and 3). In general, three quarters of respondents didn't support translation invariance, and three quarters of respondents didn't support scale invariance.

Table 1
Answers to numerical questions on scale and translation invariance in the first study by Amiel and Cowell (1992)

Amiel and Cowell 1992		Add 5 units and inequality goes…		
		Down (%)	Up (%)	Same (%)
Double income and inequality goes…	Down (%)	8	2	5
	Up (%)	15	3	17*
	Same (%)	37**	5	9

* answers in line only with translation invariance

** answers in line only with scale invariance

In grey: puzzling answers

Table 2
Comparison of percentages of answers in line with scale and translation invariance – numerical questions

Study \ Axiom	Invariance to		Scale invariance and adding a positive constant reduces inequality	Translation invariance and doubling increases inequality
	scale	translation		
Amiel and Cowell 1992	51%	31%	37%	17%
Ballano and Ruiz Castillo 1993	44%	31%	30%	19%
Harrison and Seidl 1994 (small change)	39%	31%	-	-
Harrison and Seidl 1994 (big change)	37%	18%	-	-
Jancewicz 2013	42%	55%	16%	26%
Jancewicz and Karpiński 2013	38%	49%	13%	25%

Table 3
Comparison of percentages of answers in line with scale and translation invariance – verbal questions[a]

Study \ Axiom	Invariance to		Scale invariance and adding a positive constant reduces inequality	Translation invariance and doubling increases inequality
	scale	translation		
Amiel and Cowell 1992	47%	35%	30%	17%
Ballano and Ruiz Castillo 1993	34%	30%	25%	18%
Jancewicz 2013	45%	60%	16%	20%
Jancewicz and Karpiński 2013[b]	42%	65%	13%	24%

[a] Only questions about addition of a positive constant are included: the question about subtraction of a constant was omitted for simplicity's sake. Data for all three questions can be found in: Ballano and Ruiz Castillo, 1993; Jancewicz 2013; Jancewicz and Karpiński 2013.

[b] The article analyses joint responses to three verbal questions (including the question about subtraction of a positive constant), thus percentages of supporters in the article's graphs differ from numbers presented here.

Comparing answers to verbal and numerical questions seems like a natural next step of the analysis. However Amiel and Cowell refused to perform it, pointing to numerous differences between numerical and verbal questions:

 DOI: 10.7206/DEC.1733-0092.69

- numerical and verbal questions concern different aspects of inequality
- verbal questions introduce a time sequence, since one distribution is created from the other, which is absent in numerical questions (or rather respondents are not aware of it, since researchers can create one distribution from another)
- verbal questions are suggestive and emotional, since words and phrases may convey more meaning than intended by the researchers

These arguments can be complemented by one more: if we consider a respondent to support an axiom only if they answer both the numerical and verbal questions in line with this axiom, the estimated support will fall significantly. Most researchers follow Amiel and Cowell and do not present such comparisons, however in the Polish sample (Jancewicz, Karpiński, 2013) there were only 1.4% supporters of scale and 8.5% supporters of translation invariance (both axioms and both types of questions were considered jointly). An additional problem stems from the fact that respondents often support an axiom in a verbal question, but they don't follow it when answering a numerical one – so their choices are self-contradictory (one can see how the same problem was approached by Lissowski, Tyszka and Okrasa, 1991). In such situations, we do not know whether the respondent supports the axiom or not. In general, there are both pros and cons of comparing answers to numerical and verbal questions.

Choosing one axiom over the other in case of scale and translation invariance may be especially hard for the respondents because – as shown by research relying on a different methodology – they consider both the effect of doubling incomes and the effect of adding a constant as similar (Jancewicz, 2014). One third of the respondents (36.3%), when asked to group distributions according to their income inequality, created a group consisting only of three examples: the starting distribution, the result of adding a constant, and the result of halving the incomes of the previous distribution[4]. The perceived similarity of examples representing both axioms makes it difficult to measure which rule is closer to people's definitions of inequality. It is especially hard when we consider not only the two axioms but also the middle ground between them: the possibility that we should use one axiom or the other depending on the situation (e.g. translation invariance when incomes are low and scale invariance when incomes are high) or the possibility that the compromise between the two is the appropriate rule, e.g. that dividing half of the additional money in the form of a constant addition and half in the form of multiplication of incomes is the action that leaves inequality unchanged. In general, research so far suggests that both scale and translation invariance have a grain of truth in them, though neither holds the whole truth.

[4] The distribution with an added constant and halving of its incomes were placed in the same group by 57% of respondents, while the starting distribution and the effect of adding a constant were placed together 72% of the time.

 DOI: 10.7206/DEC.1733-0092.69

To summarize, scale invariance is fulfilled by most of the popular income inequality measures such as the Gini coefficient, the Theil index, or the S80/S20. However, this axiom has the support of half of the sample when single questions are considered, and less than a quarter of the sample when questions about scale and translation invariance are considered jointly.

The Pigou-Dalton transfer principle

The Pigou-Dalton transfer principle states that transferring money from someone with a higher income to someone with a lower income reduces inequality. The amount transferred has to be small so that the high-income person and the low-income person don't switch places. There are controversies surrounding the principle. For example, Kolm (1999) argued that a transfer from a higher- to a lower-income person can, in very specific situations, increase inequality. The aforementioned Amartya Sen (1973) had doubts about whether either scale or translation invariance accurately reflected the idea of income inequality, but he was convinced that the transfer principle did. Formally the Pigou-Dalton transfer principle can be written as:

$$\forall_{i,j\in\{1..n\}} \forall_{x\in D^n} \forall_{0<t<\frac{1}{2}(x_j - x_i)} \quad (x_1, x_2, ..., x_i, ..., x_j, ..., x_n) \ >_r$$
$$(x_1, x_2, ..., (x_i + t), ..., (x_j - t), ..., x_n)$$

Support for the Pigou-Dalton transfer principle was measured in multiple studies: first, among other axioms in research done by Amiel and Cowell (1992), Ballano and Ruiz-Castillo (1993), Harrison and Seidl (1994) and Jancewicz (2013 and with Karpiński also 2013); second, in later studies that concentrated solely on the Pigou-Dalton transfer principle and its alternatives (Amiel and Cowell, 2002; Gaertner and Namazie, 2003; Amiel, Cowell and Slottje, 2004; Amiel, Cowell and Gaertner, 2012).

In the former studies, that measured support for many axioms, the Pigou-Dalton transfer principle was represented by one verbal question and two or three exemplary distributions. Answering these questions, around a half of each sample (between 49% and 60%, exactly) agreed with the Pigou-Dalton transfer principle when it was expressed verbally (the exact wording of the question is presented in Figure 2, while results from different studies are presented in Figure 3). However, numerical questions showed a completely different pattern: the percentages of answers in line with the axiom spanned from 13% to 54% (Table 4). Respondents who agreed with the transfer principle in verbal questions often answered against it (unwittingly) in the numerical question (Jancewicz, 2013, Jancewicz, Karpiński, 2013) – sometimes even in situations as extreme as the example of a transfer from the highest to the lowest income group.

 DOI: 10.7206/DEC.1733-0092.69

13. Suppose we transfer income from a person who has more income to a person who has less, without changing anyone else's income. After the transfer the person who formerly has more still has more.
    (a) *Income inequality in this society has fallen.*
    (b) *The relative position of others has also changed as a consequence of this transfer. Therefore we cannot say, a priori, how inequality has changed.*
    (c) *Neither of the above.*

***Figure 2*. Verbal question used in first research on perception of income inequality (Amiel, Cowell, 1992, Ballano, Ruiz-Castillo, 1993, Jancewicz, 2013, Jancewicz, Karpiński, 2013)**

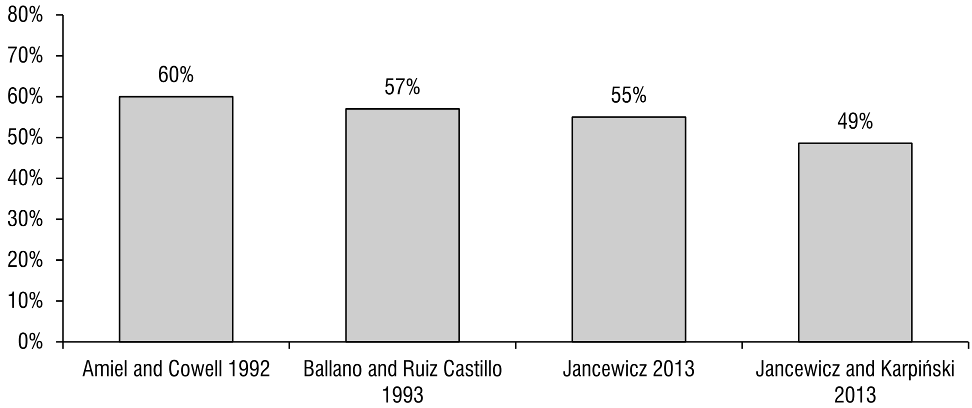

***Figure 3*. Comparison of percentages of answers in line with the Pigou-Dalton transfer principle in verbal (Figure 2 presents the exact wording of this question) – selected studies.**

Table 4
Comparison of percentages of answers in line with the Pigou-Dalton transfer principle in numerical questions depending on the example used – selected studies..

Compared distributions		Amiel and Cowell 1992	Ballano and Ruiz Castillo 1993	Harrison and Seidl 1994	Jancewicz 2013	Jancewicz and Karpiński 2013
A	B					
1, 4, 7, 10, 13	1, 5, 6, 10, 13	35%	54%		38%	37%
500, 600, 700, 800, 900	500, 640, 660, 800, 900			24%		
500, 600, 700, 800, 900	500, 600, 740, 760, 900			26%		
400, 1600, 2800, 4000, 5200	400, 2000, 2400, 4000, 5200					13%
400, 1600, 2800, 4000, 5200	800, 1600, 2400, 4000, 5200					52%

Other transfer principles

Due to the low support for the Pigou-Dalton transfer principle in numerical questions and the fact that alternative transfer principles are available, researchers

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concentrated on the problem of transfers. In the subsequent studies respondents were presented with examples of different transfers and a verbal question allowing them to decide what kind of transfer (from the higher- to the lower-income recipient) decreased inequality: every transfer (the Pigou-Dalton transfer principle); only the transfer from the recipient with the highest to the recipient with the lowest income (the Pigou transfer principle); or maybe only the transfer that leaves the ranking of incomes unchanged – the exact wording is presented in Figure 4. In the answers to this question only 15-31% of respondents expressed support for the classical Pigou-Dalton transfer principle (Figure 5).

In question 7, you are presented with a hypothetical income change and some possible views about the effects on income distribution of that change. The views are labelled (a), …, (e). Please circle the letter alongside the view that corresponds most closely to your own. You can check more than one answer, provided that you consider they do not contradict each other. Feel free to add any comment that explains the reason for your choice.

(7) Suppose income is transferred from the inhabitants of a relatively high-income region to those of a relatively low-income region, without changing the income of any other region. The transfer is not so large as to make the "rich" region "poor" and the "poor" region "rich", but it may alter their income rankings relative to the other, unaffected regions.

- *(a) The situation in Alfaland must improve if the ranking by income of all the regions remains the same. If there is any change in the income ranking of the regions then it is possible that the situations worsens or remains unaltered.*
- *(b) If the transfer is from the richest to the poorest region, and after the transfer the richest region remains the richest and the poorest remains the poorest the situation must improve. In other cases it is impossible to say a priori how the situation will change.*
- *(c) The transfer may change the relative position of other regions. So it is impossible to say a priori how the situation will change.*
- *(d) The situation in Alfaland must improve, even if there is a change in the income ranking of the regions as a result of this transfer, and even if the transfer is not from the richest region to the poorest.*
- *(e) None of the above.*

***Figure 4.* Verbal question used in research measuring support for different principles of transfers (Amiel, Cowell, 2002, Gaertner, Namazie, 2003, Amiel et al., 2004, Amiel et al., 2012).**

Note: Alfaland is an imaginary country used as a setting for the examples of income distributions. "Alfaland consists of five regions that are identical in every respect other than the incomes of their inhabitants. Everyone within a given region receives the same income, but personal incomes differ from region to region." (Amiel, Cowell and Gaertner, 2012).

Most transfer principles rely on the same idea: a transfer from the rich to the poor decreases inequality. What differs is the definitions of "poor" and "rich". The Pigou-Dalton principle considers every person with a higher income to be rich and every person with a lower income to be poor. The weak principle of transfers claims that a rich person is someone with incomes above the mean, while the poor have incomes below it. The Pigou principle says that only those with the highest income are rich, and those with the lowest income are poor. Formally these principles can be written as:

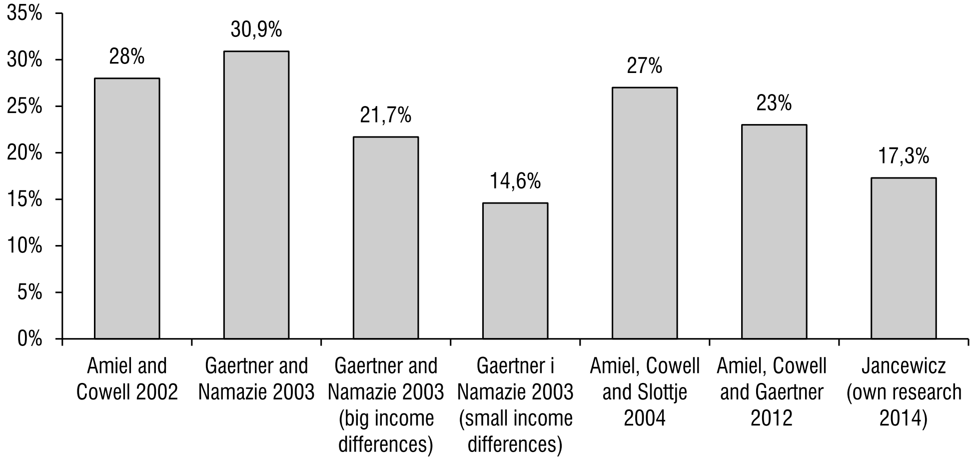

***Figure 5*. Comparison of percentages of answers in line with the Pigou-Dalton transfer principle in verbal questions – studies measuring support for different principles of transfers (Figure 4 presents the exact wording of this question).**

The Pigou transfer principle (Amiel and Cowell; 2002):

$$\forall_{x \in D^n} \forall_{0<t<\frac{1}{2}(x_n - x_1)} \quad (x_1, x_2, ..., x_n) \ >_r \ (x_1 + t, x_2, ..., x_n - t)$$

The weak principle of transfers[5] (Coulter 1989, p. 17):

$$\forall_{i,j \in \{1..n\}} \forall_{x \in D^n} \forall_{0<t<\frac{1}{2}(x_j - x_i)} \quad [x_i < \mu(x)] \ \wedge \ [x_j > \mu(x)] \ \Rightarrow$$

$$\Rightarrow (x_1, x_2, ..., x_i, ..., x_j, ..., x_n) \ >_r \ (x_1, x_2, ..., (x_i + t), ..., (x_j - t), ..., x_n)$$

Table 5 shows that respondents' opinions differ depending on the exact transfer performed. Most respondents consider a transfer from the highest- to the lowest-income group to be equalizing (54-85%). In general, the examples of transfers including income group on one of the extremes were more likely to be chosen as leading to a more equal distribution. When looking at each question separately, one could say that each transfer was considered equalizing by at least a third of the sample. However, only 10-17% of respondents consistently answered according to the Pigou-Dalton transfer principle. Interestingly, asking respondents to place the same set of examples on a scale from the least to the most unequal (instead of asking them to compare pairs) resulted in one third of the sample answering consistently in line with the Pigou-Dalton axiom (33.3%, own research conducted in 2014).

5 It is worth noting that different experts tend to call different transfer principles "weak". The definition of this principle is taken from Coulter (1989, p. 17).

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Table 5
Comparison of percentages of answers in line with the Pigou-Dalton transfer principle in the numerical questions depending on the example used – studies measuring support for different principles of transfers.

Compared distributions: A	Compared distributions: B	Amiel and Cowell 2002	Gaertner and Namazie 2003	Gaertner and Namazie 2003 (large income differences)	Gaertner and Namazie 2003 (small income differences)	Amiel, Cowell and Slottje 2004	Amiel, Cowell and Gaertner 2012	Own research 2014[a]
2, 5, 9, 20, 30	**3**, 5, 9, 20, **29**	74%	62.7%	84.8%	74.5%	54.0%	71.3%	80.0%
2, 5, 9, 20, 30	2, **6**, 9, 20, **29**	61%	56.7%	67.4%	61.7%	47.0%	59.4%	73.3%
2, 5, 9, 20, 30	2, **6**, 9, **19**, 30	48%	40.3%	47.8%	46.8%		46.2%	60.0%
2, 5, 9, 20, 30	2, **6, 8,** 20, 30	40%	32.8%	45.7%	42.7%	33.9%	37.8%	61.9%
2, 5, 9, 20, 30	2, **10**, 9, **15**, 30	60%	53.7%	45.7%	51.1%	45.4%	57.3%	64.8%
2, 5, 9, 10, 30	2, 5, 9, **20, 20**	72%	70.2%	69.6%	66.0%			
10, 10, 10, 10, 30	10, 10, 10, **20, 20**					54.4%	73.4%	75.2%
All answers are B (consistently in line with the Pigou-Dalton transfer principle):		17.0%	13.4%	10.9%	10.9%	9.8%	14.7%	33.3%

Note 1: The order of presentation is different than the one used in the questionnaires.

Note 2: No bolding was used in the questionnaires: it is used in the table for convenience of the reader.

[a] In this research, respondents were asked to place the examples on a scale instead of comparing selected pairs.

Inconsistencies in answering numerical questions about transfers are less surprising when you realise that respondents often don't see that a transfer has occurred (Jancewicz, 2013). If a respondent doesn't notice that the difference between the two presented distributions can be interpreted in terms of a transfer, they cannot include that fact in their evaluation and must rely on different characteristics of the distributions, such as its general structure. Amiel and Cowell's (1992) comparison between (1, 4, 7, 10, 13) and (1, 5, 6, 10, 13) is a good example: researchers created the second distribution by transferring 1 income unit from someone earning 7 to someone earning 4 – an equalizing transfer. However, respondents noticed a regularity in the first distribution (gaps between consecutive incomes are always 3 income units). They saw that the second distribution broke this regularity by widening the income gap between the lowest-income group and the rest of the society, thus some considered the second distribution more unequal. Kolm's (1999, p. 21) argumentation against the transfer principle was conducted in a similar manner. A transfer performed in (1, 4, 4) creates a situation (2, 3, 4) with more possibility for jealousy and everyone having a different income.

In general, the obtained results show that the Pigou-Dalton transfer principle is consistently accepted by a minority of respondents.

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4. Axioms and measures

All income inequality measures are analysed and described in terms of whether or not they fulfil the basic set of axioms. The most popular measures, such as the Gini coefficient, the Theil index and the S80/S20, fulfil the population principle and scale invariance. There are, however, axioms that differentiate these measures: the S80/S20 doesn't follow the Pigou-Dalton transfer principle, while the Gini coefficient and the Theil index do (Table 6). The Theil index distinguishes itself by allowing for decomposition of inequality by population subgroups (Foster, 1983).

Table 6
Selected income inequality measures and axioms. Axioms that are fulfilled by a measure are marked with "+" and those that aren't with "–"

Measure \ Axiom	Population principle	Scale invariance	Translation invariance	Pigou-Dalton transfer principle	Pigou transfer principle
Gini coefficient	+	+	-	+	+
Theil index	+	+	-	+	+
S80/S20	+	+	-	-	+

Research shows that only a minority of respondents evaluate the distributions (numerical questions) and axioms (verbal questions) in line with the axioms fulfilled by popular income inequality measures: population principle, scale invariance and the Pigou-Dalton transfer principle. Amiel and Cowell don't present such comparisons, but in a study (Jancewicz, 2013), replicating the first Amiel and Cowell's questionnaire experiment (1992, 1999), there were only 6.8% of answers consistently in line with all these axioms (both verbal and numerical questions were considered). If one concentrates only on answers to the numerical questions, then the corresponding percentage is 16.7%; and 12.2% when one looks only at the verbal questions.

Amiel's and Cowell's reluctance to compare answers to multiple questions is justified by the fact that categorizing someone as a supporter of an income inequality measure only if they answer a set of questions exactly in line with that measure is a dubious operation. There are better ways to interpret respondents' answers comprehensively, but they require a slightly different methodology. In the study that included placing examples on a scale from the least to the most unequal examples (own research conducted in 2014), only 3% of rankings were in line with the Gini coefficient. However, these answers and the Gini coefficient were strongly correlated (the average Pearson correlation between the Gini coefficient and respondents' answers was 0.693; Pearson correlation between the Gini coefficient and the average answer was 0.954). The examples analyzed in this study were the same as those used by Amiel, Cowell and Gaertner (2012; in their study only 14,7% of respondents consistently answered to all numerical questions in line with the Pigou-Dalton transfer principle), the only differences being the sample (Polish students) and numerical questions (placing examples on a scale instead of pairwise comparisons). This result shows that instead of concentrating on singular questions if we slightly change our methodology and look at the overall picture, we might find a different interpretation.

5. Importance of people's perceptions and multidimensionality of inequality

The differences between respondents' perceptions and income inequality measures show that neither the axioms nor the measures manage to grasp the way that ordinary people understand income inequality. In this respect, studies until today have achieved their goal: they showed existing differences. However, understanding and explaining those differences is an open question.

One can try to discredit the obtained results by arguing that there were some methodological problems like non-randomness and small sample sizes, imperfectly formulated questions or poorly chosen examples. The homogeneity of the methodology used in all the studies makes such criticism even stronger. However, the initial non-formal research by Cowell (1985), qualitative interviews (Jancewicz, 2013) and even some media interviews (often with economists) show that there is a discrepancy between how people define inequality and how income inequality measures define it.

Another line of critique is to say that the views of ordinary people, students in particular, are unimportant and should have no impact on the way the experts define inequality. This argument is often based on the assumption that the results of economic analysis are only meant for people who can thoroughly understand it and who know and accept the definitions used by scholars. It is a tempting view, since it allows one to include only the opinions of people they choose to consider an expert and to ignore the opinions of everyone else.

Taking the obtained results seriously requires an effort to understand the reasons behind the discovered discrepancies in the context of already existing knowledge. The already mentioned Amartya Sen promotes understanding inequality as a partial order, saying that *"The very notion of inequality seems to have this quasi-ordering framework. The concept is not geared to making fine distinctions and comes into its own with sharper contrast."* (1973, s. 75). Therefore, asking respondents to evaluate similar distributions will result in numerous contradictory and inconsistent answers. Only answers on distinctly different distributions will reflect the views

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of the respondents. Making the differences between distributions greater would facilitate the evaluation, increasing the consistency of answers. Some results show that in such situations respondents' answers do change. Harrison and Seidl noticed fewer answers in line with translation invariance when the positive constant added to all incomes was bigger. In a similar experiment Gaertner and Namazie (2003) compared different versions of the questionnaire: all versions had the same numbers, but the introductions differed. One of the versions made no suggestions, another suggested that the income gap between the high- and the low-income group was large, while the third one suggested that this income gap was small. Answers to the version suggesting large income differences were more often in line with the Pigou-Dalton transfer principle than they were in other versions of the experiment. In general, greater differences between examples led to answers more in line with income inequality measurement axioms.

The low support for the Pigou-Dalton transfer principle is an interesting problem on its own. We might interpret the low support of this axiom as a result of small differences between distributions, but still it looks like there is a room for improvement. Amiel and Cowell (2002) suggest choosing one of the weaker transfer axioms, like the Pigou principle of transfers, because it has a higher support among the respondents. It is a reasonable solution if our aim is to define inequality in terms of transfers. However, if our main aim is to find a measure that reflects respondents' perception of income inequality, we need to study whether the low support stems from small differences between the income distributions presented, or from the fact, that people perceive those distributions differently. It seems that many respondents do not see or look for transfers, thus their evaluations are often based on different characteristics that still need to be identified. However, respondent's opinions might change when they do actually notice the transfer and that poses another interpretational problem.

Another suggestion of Amartya Sen that helps to understand such answers is that inequality is multidimensional in its nature: *"(…) the concept of inequality has different facets which may point in different directions, and sometimes a total ranking cannot be expected to emerge."* (1973, p. 48). We can interpret Kolm's (1999) remark about the principle of transfers in a similar manner: evaluation of inequality levels can be performed from different perspectives. Kolm's (1999) example of comparing (1, 4, 4) with (2, 3, 4), might cause different reactions depending on that which aspect of income distribution one considers essential. Thus, respondents' answers can be in conflict with income inequality axioms, or even self-contradictory, if they try to simultaneously incorporate several incompatible points of view.

Accepting the notion that income inequality is a multidimensional concept creates a need to find different dimensions of inequality. Identifying the most important aspects of income inequality is difficult, for they might depend on many factors like country, culture or respondents' personal characteristics. Results are also incomparable when using different samples and methodologies. For example respondents in qualitative interviews expressed concern for those with the lowest income (Jancewicz, 2013), while a grouping task performed in a different study suggested that the existence of groups with the same income and the size of the biggest income gap are the most important factors (Jancewicz, 2014). However, these factors seemed to have little influence on respondents placing transfer examples on a scale (own research conducted in 2014). Additionally, the same set of answers can be interpreted in different ways: when studying the transfer principle, Amiel and Cowell (2002) attributed the most importance to the fact that extremes of the distribution are taking part in the transfer, while Gaertner and Namazie (2003) stressed the significance of income difference between the transfer recipient and the donor. Further research is necessary to identify and confirm which characteristics of income distributions are the most important to respondents' evaluations of income inequality.

Assuming that inequality is multidimensional requires a different approach to measuring both inequality and its perception. The methodology of eliciting respondents' views about inequality should allow for expressing this multidimensionality. Methods of analysis should be chosen to help us understand more than one dimension and possibly look at the whole pattern of answers. The changes can be small: asking respondents to place distribution on a scale, or grouping them instead of comparing selected pairs. The data obtained could be used by a multidimensional scaling algorithm to create a multidimensional map of respondents' answers (this method is described in detail in Borg and Groenen, 2005). Such an operation would let us isolate the characteristics of distributions that affect respondents' evaluations. Detecting which aspects of inequality influence people's perception the most would allow us to choose a set of income inequality measures reflecting those aspects in the best way.

Experts analyzing income inequality often use and report a whole set of income inequality measures – in a sense, their analysis is already multidimensional. However, such a set often includes measures that satisfy basic axioms, so they represent similar definitions of inequality. If we consider inequality to be multidimensional, then we should try to identify those dimensions and create a set of measures that reflect them, preferably so that each dimension is represented by a different measure. This solution can be applied to terms such as inequality, welfare or justice, but it is somewhat extensive. Since there is no universal definition, we consider a whole set. Such analysis will detect situations where different definitions are in conflict, which can make it difficult to come to a decision, but it makes the picture more complete. In general, it is an important and interesting avenue of thought that requires further research and consideration on what we mean by income inequality.

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