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Misvaluation and the dynamics of bilateral trading

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Abstract

Subjective biases and errors systematically affect market equilibria, whether at the population level or in bilateral trading. Here, we consider the possibility that an agent engaged in bilateral trading is mistaken about her own valuation of the good she expects to trade, that has not been explicitly incorporated into the existing bilateral trade literature. Although it may sound paradoxical that a subjective private valuation is something an agent can be mistaken about, as it is up to her to fix it, we consider the case in which that agent, seller or buyer, consciously or not, given the structure of a market, a type of good, and a temporary lack of information, may arrive at an erroneous valuation. The typical context through which this possibility may arise is in relation with so-called experience goods, which are sold while all their intrinsic qualities are still unknown (such as untasted bottled fine wines). We model this "private misvaluation" phenomenon in our study. The agents may also be mistaken about how their exchange counterparties are themselves mistaken. Formally, they attribute a certain margin of error to the other agent, which can differ from the actual way that another agent misvalues the good under consideration. This can constitute the source of a second-order misvaluation. We model different attitudes and situations in which agents face unexpected signals from.

Keywords: bilateral trading, misevaluation, bargaining.

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1. Introduction

It is well known, since the seminal work of Chatterjee and Samuelson (1983) (and also Myerson & Satterthwaite, 1983), that bilateral trade under circumstances of incomplete information cannot yield at equilibrium all the benefits from trade. We will keep to this basic mechanism in order to study the efficiency effect of private misvaluation.

We model the situation where sellers or buyers in a bilateral trading setting can be mistaken about the value of the good they aim to exchange, introducing a source of valuation error that has – to our best knowledge – not been previously considered in the bilateral trade literature. At first glance it may be counter-intuitive to allow for misvaluation in this context because agents can freely decide the value they assign to the goods they intend to sell or buy. Value being a purely subjective factor is regarded in principle as being immune to misidentification. We consider the cases in which private valuations may be mistaken in certain ways and how such misvaluation can also affect exchange equilibria.

There is also an obvious sense in which we may be mistaken in our subjective private valuations. We can learn that others do not value the same good in the same way, and may decide to change our initial valuation and consider that it was inappropriate to begin with. In this instance we have learnt to value that good differently and retrospectively consider that we should have valued the good the way we do now. But we can also conceive of a revision of value that does not necessarily involve the influence of other individuals. Having acquired a good at a certain price, which was in part fixed due to our way of valuing it, we may experience some regret after having actually consumed that good. We do not merely regret the price we paid, we also realise through the experience of regret that we have misvalued the good. This is probably the most common situation in which it may make sense to point to a phenomenon of private misvaluation.

In this article, we are going to model the consequences on bilateral trading price equilibria of the presence of agents that can change their private valuations. Note that the basic behavioural phenomena to which we point here is related to – yet different from – a recent attempt to consider the influence of minimax regret agents on equilibrium convergence within large markets (Shafer, 2020). Minimax regret agents can be modelled as entertaining a set of priors on future trading prices and adopt a strategy that maximises the worst-case scenarios through which the market can stabilise. In large markets this can prevent them from converging towards an agreed price. In our model, by contrast, we model the retrospective effect of a quoted price, a placed bid or offer, on consumption regret and the realisation of an initial misvaluation. This phenomenon is all the more frequent in the context of experience goods trading, i.e. goods which requires first-hand experience to be properly valued and yet must give rise to value estimation at the moment of trading (Hutter, 2011). We can think of wine-trading and suppose that at the moment of the exchange the objective quality of the wine is still partially unknown and no subjective appraisal has occurred (the bottle has not been opened and the wine tasted, see (Bourgeois-Gironde & Czupryna, 2020).

This means that if the buyers had further information on the real quality of the wine, which should be the basis for the valuation they would tend to change their valuation. This sort of situation may seem to justify the resort to informants or intermediaries in order to mitigate the adverse effects of the lack or asymmetry of information. However, this does not necessarily solve the problem. For instance, in studying the New Orleans real estate market, Zhu and Pace (Zhu & Pace, 2012), compare the appraisals of property values that are performed by money lenders and money borrowers, as well as by neutral referees. They expectedly show that, on average, lenders's appraisals are higher and borrowers's appraisals are lower than those performed by referees. The market can in principal adjust for these discrepancies, but the role of the referees in this context may in fact deviate the trading price from what its equilibrium could be by letting a free adjustment take place under conditions of symmetric information. This is due to the fact that the referees are themselves more or less experienced, which entails a regressive effect about how the borrowers or lenders should revise their own initial valuation of the property with respect to this appraisal. Misvaluation therefore appears to be a potentially widespread phenomenon in bilateral bargaining situations that take place under conditions of asymmetric information. In the labour market a worker may have overconfidence in her abilities, which a hirer cannot distinguish from genuine skills. But such misvaluation of one's abilities by the worker can make the principal better and the agent worse-off. The principal can exploit overconfident beliefs and pay a lower wage if overconfidence is a complement rather than a substitute to actual effort. On the other hand, if effort and overconfidence are substitutes, both the agent and the principal will be worse-off. (Santos-Pinto & Rosa, 2020). This example is important because we cannot exclude the possibility that the agent is genuinely mistaken about her own value on the labour market. When she realises the mismatch between how she can actually perform as a worker and the actual conditions of the job, she may oscillate between two attitudes which reflect what we call the first-order and second-order revision of her beliefs. At the first-order level, she may realise that she has misvalued her skills; at the second-order level, she may continue to believe that the principal does not realise her actual value; but it is most likely, as we will model, that she will partially revise her beliefs at both levels.

Such situations create a dynamic between the buyer and the seller affecting their respective beliefs about the way the other trader is likely to either misvalue the good or misquote a price. Under such circumstances it is even rational for the consumer to accept a misvaluation of the good and to anticipate regret. Riordan (Riordan, 1986) has shown that under monopolistic competition (here let us suppose that the seller holds a unique type of good, say a specific wine) where consumers cannot directly verify the product quality before purchase, they have no option other than to be influenced by observed prices (see Ali & Nauges, 2007) and to distrust those prices, since the lack of adequate information generates an excessive variation in product quality.

Two subsequent phenomena can occur. Firstly, as we have suggested, traders can also estimate that their trading partner actually misvalues the good in different directions (undervaluation or overvaluation). Each of the parties may then attribute to the other an interval of first-order misvaluation, whose attribution defines what we label as second-order misvaluation. Secondly, when attempts at trading occur, the traders can react in different ways to the signal they receive and to the extent with which that signal differs from their own private value. Even if they think they are liable to misvaluation, they have no particular reason to think – due to the possibility of second-order misvaluation – that they should fully incorporate the partner's signal into the revision of their own private value. They tend to distribute the revision process by conferring different weights to the reassessment of first-order and second-order beliefs. In the formulation of the model which we present in section 2, this amounts to assigning more or less strength to the bargaining power of one's quoted price (stated bid or stated offer for the buyer and seller respectively) with respect to the trading partner's one.

The agents are therefore facing a situation in which they can revise their second-order or first-order value of the good. In particular we model the case for which the quoted bid or ask price by the trading partner lies outside the interval which the agent has anticipated. We consider that in this case the agents revise their first- and second-order beliefs accordingly. The revision process may simultaneously concern two economic variables, namely first-order and second-order valuation. Therefore, we consider the trading process as a multistage process in which not only private values may influence quoted bids or ask prices, but also where the reverse effects of quoted prices on valuations are considered. That decomposition of the revision process in such multiple stages has recently been independently analysed in epistemic logic (Yuan, 2021). It has not, to our knowledge, been so in economics. The reason for this is that value and price are not supposed to diverge at equilibrium, price being a coordination of expectations between stable values. In contrast our model allows for a partial decorrelation between price and value in the particular sense that quoted prices can influence changes in valuations.

The paper is structured as follows. Section 2 analyses a simple bilateral trading model with misvaluation. Section 3 extends this baseline framework to the population level by means of an agent-based model, allowing us to study the resulting population dynamics. Section 4 concludes.

2. Bilateral trading

We use the same framework as in (Chatterjee & Samuelson, 1983), which we briefly present in the following section. The original framework and, consequently, the model developed in this paper, constitutes an example of the static game of incomplete information with the (one price linear) Bayesian-Nash equilibrium. Specifically, it admits bargaining behaviour in a bilateral setting, where a single seller of an indivisible good negotiates with a single potential buyer. A trade is concluded only if the transaction price lies between the seller's and buyer's respective reservation prices. Each agent knows their own value (reservation price) but has an incomplete information about the opponent's valuation, which is modelled through commonly known, strictly increasing and differentiable distribution functions. These distributions are common knowledge in the sense that each party is aware of both the opponent's beliefs and the fact that these beliefs are mutually recognised.

Bargaining takes place through a simple sealed-bid mechanism in which the seller and buyer simultaneously submit offers. If the buyer's bid exceeds the seller's ask, trade occurs and the price is determined as a weighted average of the two offers. In particular, the final price is set based on the buyer bid price and seller offer price p using the formula $p = k \times b + (1 - k) \times s$, for $k \in [0,1]$. The parameter k may be interpreted as bargaining power[3]. Otherwise, no transaction takes place. Within this framework, bargaining behaviour depends on each

[3] To retain generality and allow direct comparison with the original model, we maintain the parameter k.

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agent's reservation price, their beliefs about the opponent's valuation, and their expectations regarding the opponent's beliefs.

In our paper, we only consider the special case and assume that value for a seller v_s is uniformly distributed over the interval $[\underline{v}_s, \overline{v}_s]$. Analogously, the value for a buyer v_b is uniformly distributed over the interval $[\underline{v}_b, \overline{v}_b]$. Furthermore, we assume that the maximum possible value for a buyer is higher than the minimum possible value for a seller, $\overline{v}_b > \underline{v}_s$. Otherwise, no trading would be possible.

2.1. Misvaluation as overvaluation and undervaluation

We now assume 4 that both buyers and sellers may experience some private misvaluation of the good to be exchanged. We therefore consider that the value for a seller experiencing private misvaluation is modified by adding the constant δ_s, which represents the level of overvaluation (if positive) or undervaluation (if negative). As a consequence, $v_s + \delta_s$ is uniformly distributed over the interval $[\underline{v}_s + \delta_s, \overline{v}_s + \delta_s]$. Analogously, the value for a misvaluing buyer $v_b + \delta_b$ is uniformly distributed over the interval $[\underline{v}_b + \delta_b, \overline{v}_b + \delta_b]$. As explained above, we also consider that sellers and buyers can make predictions about the degree to which their partner succumbs to private misvaluation. In doing so they are liable to amplify or underestimate that phenomenon. This possibility defines what we label as second-order misvaluation. Formally, we introduce the constant δ_s' as a parameter that represents how a seller estimates the buyer's misvaluation. In particular, a seller thinks that the buyer's valuation is uniformly distributed over the interval $[\underline{v}_b + \delta_s', \overline{v}_b + \delta_s']$. Correspondingly, the buyer thinks that the seller's valuations are uniformly distributed over the interval $[\underline{v}_s + \delta_b', \overline{v}_s + \delta_b']$.

We may thus define the following measures:

1. δ_s, the seller's misvaluation level
2. δ_b, the buyer's misvaluation level
3. $\delta_s' - \delta_b$, the difference between a buyer's misvaluation level as estimated by a seller and the actual misvaluation level of a buyer – which can be defined as the seller's objective second-order misvaluation level
4. $\delta_b' - \delta_s$, which, reciprocally, is the difference between a seller's misvaluation level as estimated by a buyer and the actual misvaluation level of a seller – which refers to the buyer's objective second-order misvaluation level
5. $\delta_s' - \delta_s$, the difference between the misvaluation level of a buyer estimated by a seller and a seller's own misvaluation level – in other terms, the seller's subjective second-order misvaluation level
6. $\delta_b' - \delta_b$, the difference between the misvaluation level of a seller estimated by a buyer and a buyer's own misvaluation level – in other terms, the buyer's subjective second-order misvaluation level

Inspired by Squintani's paper (see Squintani, 2006), we firstly assume that both seller and buyer may be mistaken but do not realise it. Therefore, each of them will play according to the strategy defined in Section 2.2. The players play according to their their own and their trading partners' value distributions. In the original paper, Squintani considers two classes

[4] The preliminary results are published in Bourgeois-Gironde and Czupryna (2022).

of equilibria: naive and sophisticated. For the first class of equilibria, it is assumed that the players play rationally by simply adapting to the observed strategy of the trading partners without inferring consequences from these observations about their true valuation levels and beliefs. Both the revision of the players' beliefs and the subsequent adaptation of behaviour is considered in the sophisticated equilibrium class. We will consider only the second class of the equilibria in our paper.

2.2. Trading equilibrium

The derivation of the formulas follows an approach of the original article, taking into account a parallel shift in the distributions representing the agents' misvaluations. We assume that the strategies of both agents are linear functions of their respective original values. For each agent, we then determine the parameter values of these functions that maximise expected profits, defined as the difference between the eventual transaction price and the agent's intrinsic value when trade occurs and zero otherwise. We also consider, as in the original article, the boundary cases.

From the buyer's perspective, two situations must be considered:

- **High valuation:** The buyer's maximum bid should not exceed the highest possible selling price. Bidding higher does not increase the probability of a transaction but leads to a higher purchase price and lower profit.
- **Low valuation:** If the buyer's reservation price is below the minimum selling price, there is no incentive to submit a truthful offer, as the transaction would not take place anyway.

From the seller's perspective, two situations must also be considered:

- **High valuation:** If the seller's reservation price exceeds the maximum possible buying price, there is no incentive to submit a truthful offer, as the transaction would not take place anyway.
- **Low valuation:** The seller's minimum ask should not fall below the lowest possible buying price. Asking below this level does not increase the probability of a transaction but results in a lower sale price and reduced profit.

We therefore limit ourselves to presenting only the final results. The equilibrium offer strategy of the seller is defined as follows. For v_s satisfying the condition given in Eq. 1:

$$v_s < \frac{2-k}{1+k}\underline{v}_b + \frac{k}{2}\underline{v}_s - \frac{(2-k)(1-k)}{2(1+k)}\overline{v}_b + \frac{2-k}{2}(\delta_s' - \delta_s) \tag{1}$$

the value of an offer is given in Eq. 2.

$$s(v_s) = \frac{1}{1+k}\underline{v}_b + \frac{k(1-k)}{2(1+k)}\overline{v}_b + \frac{k}{2}\underline{v}_s + \frac{2-k}{2}\delta_s' + \frac{k}{2}\delta_s \tag{2}$$

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For v_s satisfying the condition given in Eq. 3:

$$\frac{2-k}{1+k}\underline{v}_b + \frac{k}{2}\underline{v}_s - \frac{(2-k)(1-k)}{2(1+k)}\overline{v}_b + \frac{2-k}{2}(\delta_s' - \delta_s) \leq v_s \leq \frac{2-k}{2}\overline{v}_b + \frac{k}{2}\underline{v}_s + \frac{2-k}{2}(\delta_s' - \delta_s) \tag{3}$$

the offer is given in Eq. 4.

$$s(v_s) = \frac{1}{2-k}v_s + \frac{1-k}{2}\overline{v}_b + \frac{k(1-k)}{2(2-k)}\underline{v}_s + \frac{1-k}{2}\delta_s' + \frac{1+k}{2}\delta_s \tag{4}$$

And finally, for v_s satisfying the condition given in Eq. 5:

$$v_s > \frac{2-k}{2}\overline{v}_b + \frac{k}{2}\underline{v}_s + \frac{2-k}{2}(\delta_s' - \delta_s) \tag{5}$$

the offer is given in Eq. 6.

$$s(v_s) \geq \frac{1}{2-k}v_s + \frac{1-k}{2}\overline{v}_b + \frac{k(1-k)}{2(2-k)}\underline{v}_s + \frac{1-k}{2}\delta_s' + \frac{1+k}{2}\delta_s \tag{6}$$

Analogously, the strategy for a buyer is presented below. For v_b satisfying the condition given in Eq. 7:

$$v_b \leq \frac{1-k}{2}\overline{v}_b + \frac{1+k}{2}\underline{v}_s + \frac{1+k}{2}(\delta_b' - \delta_b) \tag{7}$$

the value of a bid is given in Eq. 8:

$$b(v_b) \leq \frac{1}{1+k}v_b + \frac{k(1-k)}{2(1+k)}\overline{v}_b + \frac{k}{2}\underline{v}_s + \frac{k}{2}\delta_b' + \frac{2-k}{2}\delta_b \tag{8}$$

For v_s satisfying the condition given in Eq. 9:

$$\frac{1-k}{2}\overline{v}_b + \frac{1+k}{2}\underline{v}_s + \frac{1+k}{2}(\delta_b' - \delta_b) \leq v_b \leq \frac{1+k}{2-k}\overline{v}_s + \frac{1-k}{2}\overline{v}_b - \frac{k(k+1)}{2(2-k)}\underline{v}_s + \frac{1+k}{2}(\delta_b' - \delta_b) \tag{9}$$

the bid is given in Eq. 10:

$$b(v_b) = \frac{1}{1+k}v_b + \frac{k(1-k)}{2(1+k)}\overline{v}_b + \frac{k}{2}\underline{v}_s + \frac{k}{2}\delta_b' + \frac{2-k}{2}\delta_b \tag{10}$$

And finally, for v_b satisfying the condition given in Eq. 11:

$$v_b > \frac{1+k}{2-k}\overline{v}_s + \frac{1-k}{2}\overline{v}_b - \frac{k(k+1)}{2(2-k)}\underline{v}_s + \frac{1+k}{2}(\delta_b' - \delta_b) \tag{11}$$

the bid is given in Eq. 12:

$$b(v_b) = \frac{1}{2-k}\overline{v}_s + \frac{1-k}{2}\overline{v}_b + \frac{k(1-k)}{2(2-k)}\underline{v}_s + \frac{1+k}{2}\delta_b' + \frac{1-k}{2}\delta_b \tag{12}$$

Let us now consider as an example, the differences between expected and quoted price, first by a seller and then by a buyer. Namely, the seller will trade the following price, see Eq. 13:

$$s(v_s) = \frac{1}{2-k}(v_s + \delta_s) + \frac{1-k}{2}(\overline{v}_b + \delta_s') + \frac{k(1-k)}{2(2-k)}(\underline{v}_s + \delta_s) \tag{13}$$

However, the buyer thinks that the seller would trade the following price, see Eq. 14:

$$b'(v_s) = \frac{1}{2-k}(v_s + \delta_b') + \frac{1-k}{2}(\overline{v}_b + \delta_b) + \frac{k(1-k)}{2(2-k)}(\underline{v}_s + \delta_b') \tag{14}$$

Thus, the difference between the seller's real offer and the buyer's expectations can be expressed using the formula in Eq. 15 and is the weighted sum of the seller's and buyer's second order confidence levels.

$$s(v_s) - b'(v_s) = \frac{1-k}{2}(\delta_s' - \delta_b) + \frac{k+1}{2}(\delta_s - \delta_b') \tag{15}$$

Similarly, the difference between the buyer's real offer and the seller's expectations can be expressed using the formula in Eq. 16 and is the weighted sum of the seller's and buyer's second-order confidence levels.

$$b(v_b) - s'(v_b) = \frac{2-k}{2}(\delta_b - \delta_s') + \frac{k}{2}(\delta_b' - \delta_s) \tag{16}$$

2.3. Trading dynamics

We have already assumed that both a seller and a buyer will play according to the strategy defined in Eqs. 1–12, in line with their beliefs about their own value distributions and those of their trading partners. We now analyse the case where the agents may realise that their assumptions about their valuation or their expectations about the valuation distributions of the trading partner may be wrong. This will form the basis for a mechanism that will be employed in the multi-agent based model presented in the following section, during the agents' learning process.

We will consider four possible cases of misvaluation:

- a buyer quotes a higher price than the price expected by a seller
- a buyer quotes a lower price than the price expected by a seller
- a seller quotes a lower price than the price expected by a buyer
- a seller quotes a higher price than the price expected by a buyer

Let us also assume that buyers and sellers believe that the ranges containing potential values for buyers and sellers intersect. This assumption leads to the following conditions 17 and 18 being satisfied:

$$\overline{v}_b - \underline{v}_s > \delta_s - \delta_s' > \underline{v}_b - \overline{v}_s \tag{17}$$

$$\overline{v}_b - \underline{v}_s > \delta_b' - \delta_b > \underline{v}_b - \overline{v}_s \tag{18}$$

These assumptions are not excessively restrictive and simplify the analysis of the dynamics.

2.3.0.1. Buyers quote a higher price than that expected by sellers

Sellers firstly consider which of the two potential prices – all buyers' or all sellers' maximum prices (these are the maximum prices offered by any buyer or seller respectively) – is higher based on their beliefs. The former is satisfied when the condition given in Eq. 19 is met.

$$\overline{v}_b - \frac{2}{2-k}\overline{v}_s + \frac{k}{2-k}\underline{v}_s \geq \delta_s - \delta_s' \tag{19}$$

In this case, a seller believes that a buyer should not quote a price higher than what the buyer considers as the possible maximum sell price quoted by any seller 5.

Let us now consider the situation where the condition given in Eq. 19 is not met. In this event, a seller believes that a buyer should not quote a price higher than the possible maximum buy price quoted by any buyer 6.

Let Δ denote the difference between the actual bid price and the bid price expected by a seller. Sellers will increase their offers by Δ without changing the values of δ_s and δ_s' when they use the naive updating strategy. On the other hand, sellers will increase the values of δ_s and δ_s' by Δ when they use the sophisticated strategy. It is also worth mentioning that both updating strategies, the naive and sophisticated, lead to the sellers' offer prices increasing by Δ, yet also lead to differences in situations when the boundary cases (discussed in Section 2.2) are applied. The updating process in the following paragraphs is analogous to the one described here.

2.3.0.2. Buyer quotes a lower price than the price expected by seller

Sellers firstly consider which of the two potential prices – all buyers' or all sellers' minimum prices – is lower, based on their beliefs. The former is satisfied when the condition given in Eq. 20 is met.

$$\underline{v}_s + \frac{1-k}{1+k}\overline{v}_b - \frac{2}{1+k}\underline{v}_b > \delta_s' - \delta_s \tag{20}$$

In this case, a seller knows that a buyer may bid any price (see Eq. 8) and does not update their strategy. Now consider the situation where the condition given in Eq. 20 is not met. In this event, a seller knows that a buyer should not quote a price lower than the possible minimum buy price quoted by any buyer 7.

[5] This is the price, from a seller's perspective, that would be quoted by a seller having a maximum possible private value. The value can be calculated by substituting a value v_s for a variable $\overline{v}_s$ in Eq. 4.

[6] This is the price, from a seller's perspective, that would be quoted by a buyer having a maximum possible private value. The value can be calculated by substituting a value v_b for a variable $\overline{v}_b$ in Eq. 10 and replacing δ_b with δ_s' and δ_b' with δ_s.

[7] This is the price, from a seller's perspective, that would be quoted by a buyer having a minimum possible private value. The value can be calculated by substituting a value $\underline{v}_b$ for a variable v_b in Eq. 10 and replacing δ_b with δ_s' and δ_b' with δ_s.

2.3.0.3. Seller quotes a higher price than the price expected by buyer

Buyers firstly consider which of the two potential prices – all buyers' or all sellers' maximum prices – is higher, based on their beliefs. The former is satisfied when the condition given in Eq. 21 is met.

$$\overline{v}_b - \frac{2}{2-k}\overline{v}_s + \frac{k}{2-k}\underline{v}_s \geq \delta_b' - \delta_b \tag{21}$$

Here a buyer knows that a seller should not quote a price higher than the possible maximum sell price quoted by any seller. When the condition given in Eq. 21 s not met, a buyer knows that a seller may bid any price (see Eq. 6) and does not update.

2.3.0.4. Seller quotes a lower price than the price expected by buyer

Buyers firstly consider which of the two potential prices – all buyers' or all sellers' maximum prices – is lower, based on their beliefs. The former is satisfied when the condition given in Eq. 22 is met.

$$\underline{v}_s + \frac{1-k}{1+k}\overline{v}_b - \frac{2}{1+k}\underline{v}_b > \delta_b - \delta_b' \tag{22}$$

In this case, a buyer believes that a seller should not quote a price lower than the possible minimum sell price quoted by any seller. In the event that the condition given in Eq. 22 is not met, the buyer believes that the seller should not quote a price lower than what the seller considers as the possible minimum buy price quoted by any buyer.

2.4. Simple misvaluation levels' dynamics

In this section we analyse the dynamics of the misvaluation levels in a more detailed way. We will first analyse the situation when the quoted price is higher than expected. To simplify the analysis we only consider boundary cases and assume that $v_s = \overline{v}_s$ and $v_b = \overline{v}_b$. In such circumstances both the buyer and seller will quote the highest possible price which can be quoted by all buyers, or all sellers, respectively.

Case A. Let us assume that the following conditions are satisfied:

$$\overline{v}_b - \frac{2}{2-k}\overline{v}_s + \frac{k}{2-k}\underline{v}_s \geq \delta_b' - \delta_b \tag{23}$$

$$\overline{v}_b - \frac{2}{2-k}\overline{v}_s + \frac{k}{2-k}\underline{v}_s \geq \delta_s - \delta_s' \tag{24}$$

In this case, both the buyer and seller think that the potential all buyers' maximum price is higher than the potential all sellers' maximum price. As a consequence, the buyer's bid price is given by Eq. 12 and the seller's offer price is given by Eq. 4 when $v_s = \overline{v}_s$.

The process of updating the misvaluation levels depends on the sign of the following expression (see Eq. 25):

$$\frac{1-k}{2}(\delta_s' - \delta_b) + \frac{1+k}{2}(\delta_s - \delta_b') \tag{25}$$

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If the sign is positive (an actual seller's offer price is higher than the price expected by a buyer) the buyer will increase δ_b and δ_b' so that the value of the expression 25 becomes zero. Otherwise, if the sign is negative (an actual buyer's bid price is higher than the price expected by a seller) the sellers will increase δ_s and δ_s' in a similar way.

Case B. Let us assume that the condition given in Eq. 23 is satisfied but the one given in Eq. 24 is not. In this case, only the buyer believes that the potential all buyers' maximum bid price is higher than the potential all sellers' maximum offer price. As a result, the seller's offer price is given by Eq. 6 and the buyer believes that the seller's offer price is given by Eq. 4 for $v_s = \overline{v}_s$. When the condition given in Eq. 26 is satisfied, the buyer will increase δ_b and δ_b' similarly to case A.

$$\frac{1-k}{2}(\delta_s' - \delta_b) + \frac{k+1}{2}(\delta_s - \delta_b') + \epsilon > 0 \tag{26}$$

Moreover, the buyer's bid price in this case is given by Eq. 12, whereas the seller thinks that the buyer's bid price is given by Eq. 10 for . The buyer will quote a higher bid price than the price expected by a seller when the following condition is satisfied:

$$\frac{1}{2-k}\overline{v}_s - \frac{1}{2}\overline{v}_b - \frac{k}{2(2-k)}\underline{v}_s + \frac{1+k}{2}\delta_b' + \frac{1-k}{2}\delta_b - \frac{k}{2}\delta_s - \frac{2-k}{k}\delta_s' > 0 \tag{27}$$

We know that the sum of the first three components of Eq. 27 must be higher than $\frac{1}{2}(\delta_s' - \delta_s)$ and lower than or equal to $\frac{1}{2}(\delta_b - \delta_b')$. By inserting the lower bound value into Eq. 27, we obtain the condition given in Eq. 28; by plugging in the upper bound value, we get the condition given in Eq. 29.

$$\frac{1-k}{2}(\delta_b - \delta_s') + \frac{1+k}{2}(\delta_b' - \delta_s) > 0 \tag{28}$$

$$\frac{2-k}{2}(\delta_b - \delta_s') + \frac{k}{2}(\delta_b' - \delta_s) > 0 \tag{29}$$

Let us now assume that the conditions given in Eq. 26 and Eq. 29 are simultaneously satisfied. In this situation, buyers will increase their misvaluation levels (δ_b and δ_b') to reduce the value of the condition given in Eq. 26 to zero and the sellers will simultaneously increase their misvaluation levels (δ_s and δ_s') to reduce the value of the condition given in Eq. 29 to zero.

Let us set ϵ[8] to zero to simplify the analysis. Let us also denote the value of the condition given in Eq. 26 as x and the value of the condition given in Eq. 29 as y. Both x and y are positive. In this case, the levels of δ_b and δ_b' are increased by x and the levels of δ_s and δ_s' by y. After the updating of the misvaluation levels by both a buyer and a seller, the value of the condition given in Eq. 26 will be y and the value of the condition given in Eq. 29 will be x. And the traders will update their misvaluation levels once again. These simultaneous changes of misvaluation levels for both buyers and sellers leave the values of both conditions un-

[8] We use the symbol ϵ as a measure of the difference between the left and right hand expressions in Eqs 6 and 8. This refers to the situations where sellers may offer any price, provided it is high enough, and buyers may bid any price, provided it is low enough.

changed. In this special case, trading infinitely many times may cause the misvaluation levels to increase to infinity.

It is interesting to note that we may have in this particular case a specific source of bubble phenomena, owing to the possibility that both sellers and buyers symmetrically consider that, given the occurrence of quoted prices on the market, they may have initially privately misvalued the good at stake. Increasing prices on the wine market or the real estate market, both accepted by sellers and buyers (those who can continue to afford the good at those higher prices) might then be due not only to adaptation to supply and demand but also might be anchored in the fact that given the market dynamics that is entailed by revision of private valuation, both sellers and buyers start to think that these increasing prices should be indicative of the value they should genuinely attach to those goods.

Case C. Let us assume that the condition given in Eq. 23 is not satisfied but the one given in Eq. 24 is. In such a case, only the seller thinks that the potential all buyers' maximum bid price is higher than the potential all sellers' maximum offer price. Consequently, the seller's offer price is given by Eq. 4 for $v_s = \overline{v}_s$, yet the buyer believes that the seller's offer price is given by Eq. 6. Since buyers believe that sellers may offer any price provided it is high enough, buyers will not update their misvaluation levels.

Moreover, the buyer's bid price in this case is given by Eq. 10 for $v_b = \overline{v}_b$ bu the sellers think that the buyers' bid price is given by Eq. 12. The buyer will quote a higher bid price than the price expected by the seller when the following condition is satisfied:

$$\frac{1}{2}\overline{v}_b - \frac{1}{2-k}\overline{v}_s + \frac{k}{2(2-k)}\underline{v}_s + \frac{k}{2}\delta_b' + \frac{2-k}{2}\delta_b - \frac{1+k}{2}\delta_s - \frac{1-k}{k}\delta_s' > 0 \tag{30}$$

We know that the sum of the first three components of Eq. 30 must be higher than or equal to $\frac{1}{2}(\delta_s - \delta_s')$ and lower than $\frac{1}{2}(\delta_b' - \delta_b)$. By plugging the lower bound value into Eq. 30 we get the condition given in Eq 29; when plugging in the upper bound value we obtain the condition given in Eq. 28. In the former case (Eq. 29), the sellers will always update, whereas in the latter case (Eq. 28) an update is a possibility but not guaranteed.

Case D. Let us assume that both conditions given in Eq. 23 and in Eq. 24 are not satisfied. This is where both the buyer and seller believe that the potential all buyers' maximum price is lower than the potential all sellers' maximum price. Therefore, the buyer's bid price is given by Eq. 10 when $v_b = \overline{v}_b$ and the seller's offer price is given by Eq. 6. Since sellers may offer any price provided that it is high enough, buyers will not update their misvaluation levels, similarly to what occurs in case C.

The sellers will update their misvaluation levels when the condition given in Eq. 31 is satisfied.

$$\frac{k}{2}(\delta_b' - \delta_s) + \frac{2-k}{2}(\delta_b - \delta_s') > 0 \tag{31}$$

The sellers will increase δ_s and δ_s' in such a way that the value given in the left hand side of the expression in Eq. 31 becomes zero.

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We will now analyse the situation where the quoted prices are lower than expected. As for the previously described cases, to simplify the analysis we only consider boundary cases and assume that $v_s = \underline{v}_s$ and $v_b = \underline{v}_b$. In this case, both a buyer and a seller will quote the lowest possible price which can be quoted by all buyers or all sellers, respectively.

Case E. Let us assume that the following conditions are satisfied:

$$\underline{v}_s + \frac{1-k}{1+k}\overline{v}_b - \frac{2}{1+k}\underline{v}_b > \delta_b - \delta_b' \tag{32}$$

$$\underline{v}_s + \frac{1-k}{1+k}\overline{v}_b - \frac{2}{1+k}\underline{v}_b > \delta_s' - \delta_s \tag{33}$$

Here, both the buyer and seller think that the potential all buyers' minimum bid price is lower than the potential all sellers' minimum offer price. As a result, the buyer's bid price is given by Eq. 8 and the seller's offer price is given by Eq. 4 when $v_s = \underline{v}_s$.

As buyers may bid any price provided it is low enough, sellers will not update their misvaluation levels. The buyers will update their misvaluation levels when the condition given in Eq. 34 is satisfied.

$$\frac{1+k}{2}(\delta_b' - \delta_s) + \frac{k+1}{2}(\delta_b - \delta_s') > 0 \tag{34}$$

The buyers will decrease δ_b and δ_b' in such a way that the value given on the left hand side of Eq. 34 becomes zero.

Case F. Let us assume that the condition given in Eq. 32 is satisfied but the one given in Eq. 33 is not. Thus, only the buyer thinks that the potential all buyers' minimum bid price is lower than the potential all sellers' minimum offer price. Therefore, the buyer's bid price is given by Eq. 8 whereas the seller believes that the buyer's bid price is given by Eq. 10 for $v_b = \underline{v}_b$. In this event, the condition given in Eq. 35 is satisfied and the sellers will decrease δ_s and δ_s' similarly to case E.

$$\frac{2-k}{2}(\delta_s' - \delta_b) + \frac{k}{2}(\delta_s - \delta_b') + \epsilon > 0 \tag{35}$$

Moreover, in this case the seller's offer price is defined by Eq. 2 but the buyer thinks that the seller's offer price is given by Eq. 4 for $v_s = \underline{v}_s$. The seller will quote a lower offer price than what is expected by the buyer when the following condition is satisfied:

$$\frac{1}{2}\underline{v}_s - \frac{1-k}{2(1+k)}\overline{v}_b - \frac{1}{1+k}\underline{v}_b + \frac{1+k}{2}\delta_b' + \frac{1-k}{2}\delta_b - \frac{k}{2}\delta_s - \frac{2-k}{k}\delta_s' > 0 \tag{36}$$

We know that the sum of the first three components of Eq. 36 must be higher than $\frac{1}{2}(\delta_b - \delta_b')$ and lower than or equal to $\frac{1}{2}(\delta_s' - \delta_s)$. By plugging the lower bound value in Eq. 36 we obtain the condition given in Eq. 37; when plugging in the upper bound value, we get the condition given in Eq. 38.

$$(\delta_b - \delta_s') + \frac{k}{2}(\delta_b' - \delta_s) > 0 \tag{37}$$

$$\frac{1-k}{2}(\delta_b - \delta_s') + \frac{k+1}{2}(\delta_b' - \delta_s) > 0 \tag{38}$$

Now we consider the conditions given in Eq. 35 (we set ϵ to zero to simplify the analysis) and Eq. 38. In the former case, sellers will decrease their misvaluation values (δ_s and δ_s') to reduce the value of the condition given in Eq. 35 to zero, and simultaneously the buyers will decrease their misvaluation values (δ_b and δ_b') to reduce the value of the condition given in Eq. 38 to zero also. We can show – analogously to Case B – that in this special case, trading infinitely many times may cause the misvaluation levels to decrease to minus infinity, if we allow for negative prices.

Case G. Let us assume that the condition given in Eq. 32 is not satisfied but the one given in Eq. 33 is satisfied. In such a case, only the seller thinks that the potential all buyers' minimum bid price is lower than the potential all sellers' minimum offer price. As a consequence, the buyer's bid price is given by Eq. 10 for $v_b = \underline{v}_b$ but the seller believes that the buyer's bid price is given by Eq. 8. Since sellers believe that the buyers may offer any price provided it is sufficiently low, sellers will not update their misvaluation levels.

Moreover, the seller's offer price is given here by Eq. 6 for $v_s = \underline{v}_s$ but the buyers think that the seller's offer price is given by Eq. 2. The seller will quote a lower bid price than the price expected by the buyer when the following condition is satisfied:

$$-\frac{1}{2}\underline{v}_s - \frac{1-k}{2(1+k)}\overline{v}_b + \frac{1}{1+k}\underline{v}_b + \frac{k}{2}\delta_b' + \frac{2-k}{2}\delta_b - \frac{1+k}{2}\delta_s - \frac{1-k}{k}\delta_s' > 0 \tag{39}$$

We know that the sum of the first three components of Eq. 39 must be higher than or equal to $\frac{1}{2}(\delta_b' - \delta_b)$ and lower than or equal to $\frac{1}{2}(\delta_s - \delta_s')$. By plugging in the lower bound value in Eq. 39 we obtain the condition given in Eq. 38 when plugging in the upper bound value, we obtain the condition given in Eq. 37. In the former case, the buyers will always update, whereas in the latter case an update is possible but not certain.

Case H. Let us assume that both of the conditions given in Eq. 32 and in Eq. 33 are not satisfied. In this case, both the buyer and seller think that the potential all buyers' minimum bid price is higher than the potential all sellers' minimum offer price.

Consequently, the buyer's bid price is given by Eq. 10 when $v_b = \underline{v}_b$ and the seller's offer price is given by Eq. 2.

The updating process will depend on the sign of the following expression:

$$\frac{2-k}{2}(\delta_s' - \delta_b) + \frac{k}{2}(\delta_s - \delta_b') \tag{40}$$

If the sign is positive (the buyer price is lower than the price expected by a seller), the seller will decrease δ_s and δ_s' so that the value of the expression 40 becomes zero. Otherwise, if the sign is negative (the seller price is lower than the price expected by a buyer), the buyers will decrease δ_b and δ_b' in a similar way.

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3. Simulation results

3.1. Model

In the model, we generalise the framework presented in the previous theoretical section of the paper. In particular, we consider a population of agents – equal numbers of buyers and sellers – with specified reservation values and misvaluation levels. Agents are randomly paired and engage in trade, following the procedure detailed in the theoretical section. The extension of the basic model allows agents to update both their own valuations and their assessments of their trading partners' valuations (in particular by changes in their misvaluation levels), in accordance with the dynamics described in cases A through H in Section 2.4. The model can potentially be interpreted as an example of evolutionary game theory, although it does not incorporate am explicit selection mechanism.

We analyse the effects of misvaluation levels represented by the parameters δ_s, δ_s', δ_b, δ_b' on trading probability and the observed mean price by means of simulation. Agent-based simulations of various trading mechanisms to study price dynamics have been performed in the past. In particular (Pellizzari & Dal Forno, 2007) study the impact of different market protocols and types of auctions by controlling for agents' behaviour. Conversely, agent-based modelling of trading behaviour in experimental and simulated stock markets had been pursued in (Grazzini, 2013). In contrast to earlier studies, we take the approach of studying one single mechanism, bilateral trading, and explore how different types of beliefs impact the price dynamics of a unit good.

The results in certain special cases may be straightforward. For example, when all parameters assume the same value $\delta_s = \delta_s' = \delta_b = \delta_b'$ the trade probability remains unchanged. For more general results and a systematic numerical exploration of the effects of our different configurations of overconfidence on trading we used a simulation approach.

For this purpose we assume that each of the parameters δ_s, δ_s', δ_b, δ_b' can vary in the range [0,0.5] and the parameter k in the range [0.25,0.75]. For each seller the value v_s is drawn from a uniform distribution over the interval $[\gamma_s, 1 + \gamma_s]$, whereas for each buyer the value v_b is drawn from a uniform distribution over the interval $[\gamma_s + \gamma_b, 1 + \gamma_s + \gamma_b]$. Both γ_s and γ_b are the simulation parameters that may assume values in the range [0,0.75] and [0,0.5] respectively.

As defined in Eq. 6, in some special cases sellers may propose any price, provided it is high enough, and analogously in Eq. 8 buyers may propose any price, provided it is low enough. We model this by allowing differences between traded prices and the boundary prices that satisfy the previously mentioned conditions with equality[9]. The exact values of these differences are drawn – during simulation – randomly from the uniformly distributed random variable that assumes values in the range $[0, \eta]$. The parameter η may admit values within the range [0,0.05].

The model is implemented in Java, using the MASON 19 framework[10]. We represented 2000 agents in the model, divided into 1000 buyers and 1000 sellers. We also analysed 500 different parameter sets obtained by systematically searching the parameter space using the

[9] It is assumed that the bid must be less than or equal to, the ask price higher than or equal to boundary values.

[10] Marcin Czupryna (2026, January 14). "Bargaining with misvaluation" (Version 1.0.0). CoMSES Computational Model Library. Retrieved from: https://www.comses.net/codebases/d1b4b130-3da0-4187-9556-040ee395fa85/releases/1.0.0/

Sobol numbers (Bratley & Fox, 1988; Christophe & Petr, 2014). We ran 100 simulation steps for each parameter set. In each step the buyers and sellers are randomly matched in pairs and make offers (bargain). If the bid price is higher than or equal to the offer price the trade takes place. After the bargaining process the agents update their strategy and may trade again in the next simulation step.

3.2. Results

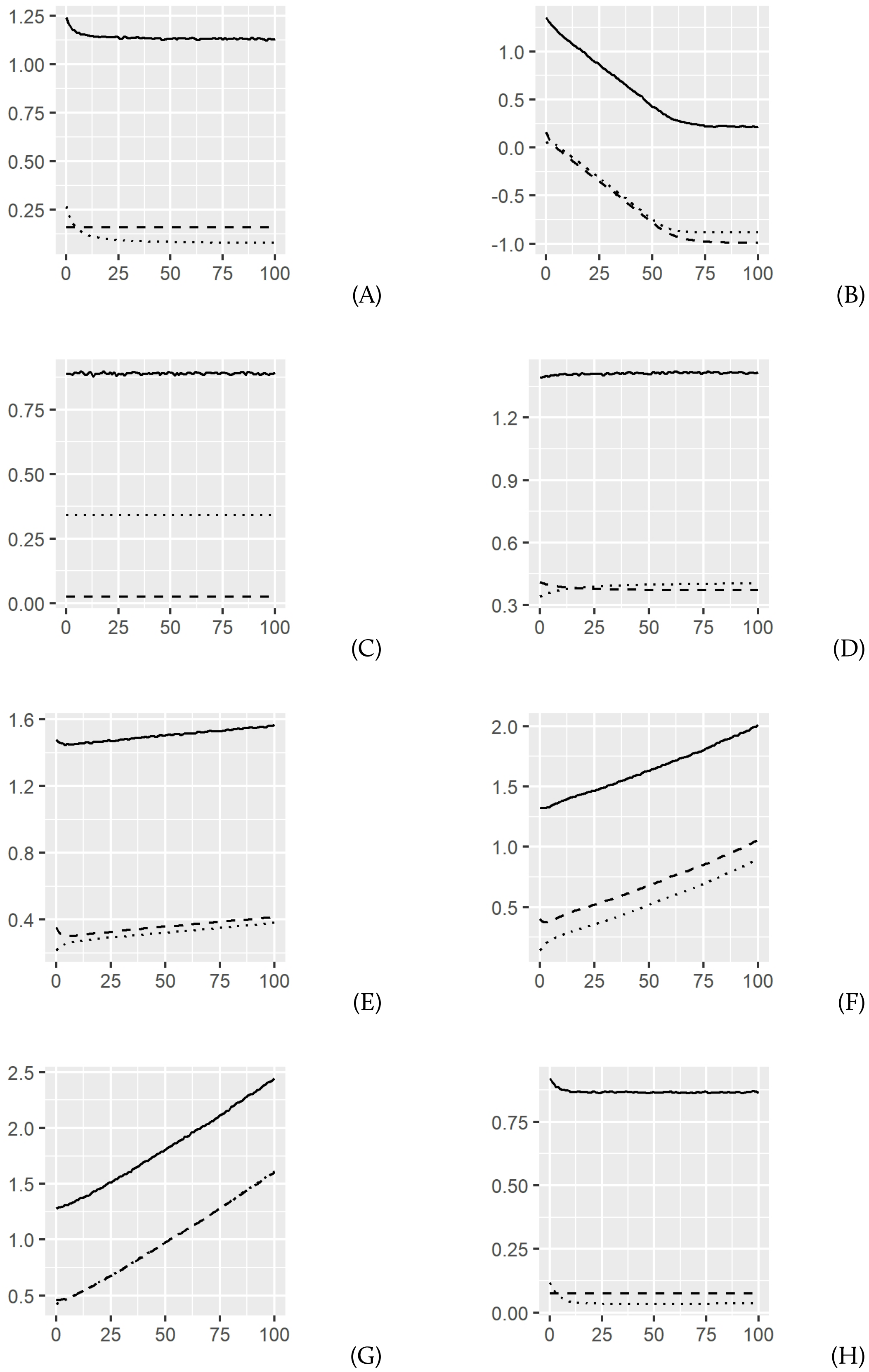
We present the sample results for 14 different scenarios selected based on the analysis presented in Section 2.4. In particular, we checked whether the conditions given in Eqs. 24, 23, 33, and 32 are satisfied or not. For each combination that has occurred (14 out of 16 potential combinations), we selected this parameter set which was characterised by the minimum value of the noise parameter η. These combinations are presented in Table 1.

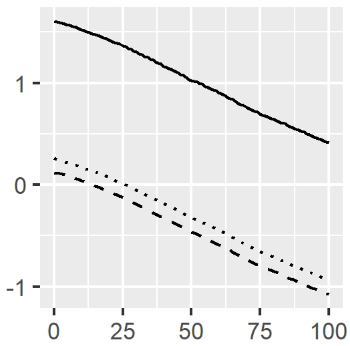
TABLE 1. The description of each variant

	Eq. 24	Eq. 23	Eq. 33	Eq. 32	Cases A–D	Cases E–H
1	FALSE	FALSE	FALSE	FALSE	D	H
2	FALSE	FALSE	FALSE	TRUE	D	F
3	FALSE	FALSE	TRUE	FALSE	D	G
4	FALSE	FALSE	TRUE	TRUE	D	E
5	FALSE	TRUE	FALSE	FALSE	B	H
6	FALSE	TRUE	TRUE	FALSE	B	G
7	FALSE	TRUE	TRUE	TRUE	B	E
8	TRUE	FALSE	FALSE	FALSE	C	H
9	TRUE	FALSE	FALSE	TRUE	C	F
10	TRUE	FALSE	TRUE	TRUE	C	E
11	TRUE	TRUE	FALSE	FALSE	A	H
12	TRUE	TRUE	FALSE	TRUE	A	F
13	TRUE	TRUE	TRUE	FALSE	A	G
14	TRUE	TRUE	TRUE	TRUE	A	E

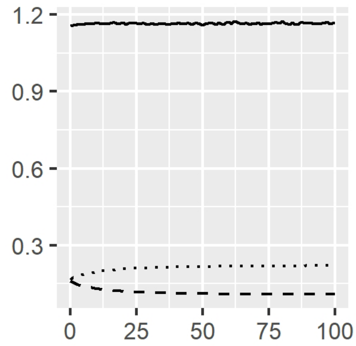
The results are presented in Figure 1. We can produce varying patterns for average prices and changes in misvaluation levels: increasing, decreasing and relatively stable. Conditions described in Case B lead to price increases, whereas conditions described in Case F lead to price decreases.

FIGURE 1. Mean price changes (solid line), mean buyers' misvaluation level (dashed line), and mean sellers' misvaluation level (dotted line) over 100 simulation steps. The numbering of subfigures corresponds to the cases listed in Table 1





(I)



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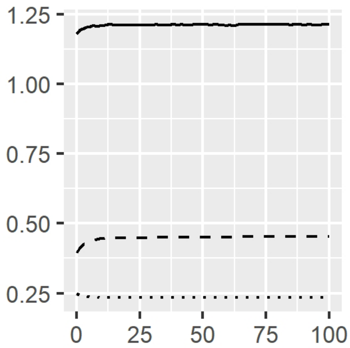
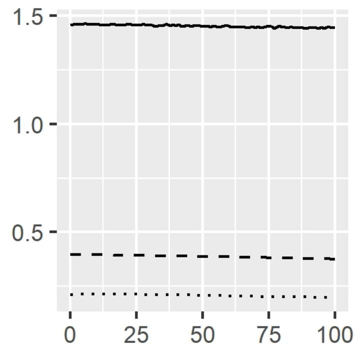
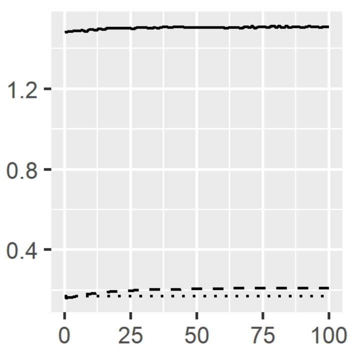


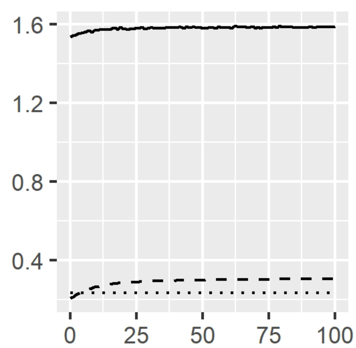
(K)



(L)



(M)



(N)

4. Conclusions

It has been well known, since the seminal work of Chatterjee and Samuelson (1983) that bilateral trade under incomplete information cannot yield at equilibrium all the benefits from trade. We stuck to this basic framework to study the effects of private misvaluation on the probability of trade and trading prices when the two agents, the seller and the buyer, can be erroneous about their own valuation of a good and about the way the other agent may commit a similar error. This phenomenon of subjective error can arise with respect to certain types of goods when the intrinsic properties of that good cannot all be ascertained at the moment of trading. Experience goods (Shapiro, 1983; Bergemann & Välimäki, 2006) provide immediate examples, for which the players can either experience consumption regret or enter into more complex belief-revision processes. For instance, their own taste can be influence by quoted prices that they attribute to agents whom they consider to have greater expertise in evaluating the good at stake. Our setting allowed for a practical definition of first-order and second-order misvaluation. In our model, agents can be biased about the ways the other agents are biased, which may count as a specific source of extended or contracted opportunities for trade.

In discussing the limits to the no-trade theorem, Gizatulina and Hellman (2019) have shown that if one of the agents assigns some small probability to the other agent being irrational, there appear certain conditions under which exchange becomes formally possible. Our agents accentuate this feature because they may also be mistaken in the way others can be mistaken. And we indeed observed that in the case of the buyers being subject to first-order misvaluation, the trading probability and price significantly increase. For the second-order misvaluation the converse applies. For sellers, on the other hand, the first order misvaluation tends to decrease the trading frequency but to increase the trading price in a statistically significant way. The second order misvaluation has precisely the opposite effect.

Interestingly, both sellers and buyers may revise their value to the same extent when facing the respective quoted bids and quoted offers by their trading partners. This specific case entails an upward (or downward) revision process. In the upward case, prices can continue to increase indefinitely, leading to the formation of price bubbles. This phenomenon is worth pointing out, since it may offer an explanation of bubble formation that is not due to speculation or to errors (see: Ackert *et al*., 2009) for this type of explanations) but is instead cognitively anchored in a symmetric belief-revision process performed by trading partners. The phenomenon is due, in our model, to the possibility to doubt one's own private valuation after receiving unexpected price signals. What we have modelled and tested, therefore, is essentially this interplay between price and value when the interaction between traders can lead them to change their initial valuation during the trading sequence. By doing so they trigger belief-revision processes that can fix final prices over equilibria which may drastically differ from the ones that would have been reached with immutable private values.

The results obtained, in particular the various patterns of price and quotation dynamics, can be compared with available empirical data from selected markets. In this way, the theoretical and simulation results presented in this article provide an (additional) potential explanation of the mechanisms underlying the observed dynamics of prices and quotations.

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5. Appendix

5.1. Advanced misvaluation levels' dynamics

So far we have assumed that the traders update their first-order and second-order misvaluation levels to the same extent. Let us now assume that this is not true. Under these circumstances traders' behaviour may change significantly during the learning process, leading to a complex dynamic arising. It is worth noting that the type of complexity introduced here, by specifying different degrees to which agents revise their beliefs at the first or second order, differs from the between-agent heterogeneity analysed in similar simulation studies of beliefs and price dynamics. The model we propose in the next section exemplifies this distinction (see the classical (Brock & Hommes, 1998), and more recently (Gong and Yang, 2020).

Here we will limit ourselves to describing a single example in a more detailed way. Consider a seller that alternately meets a buyer with maximum value $v_b = \overline{v}_b$ and a buyer with minimum value $v_b = \underline{v}_b$. Let us also assume that the conditions for Case A and Case H are simultaneously satisfied.

A seller must select pairs of first-order and second-order misvaluation levels, such that the following conditions given in Eqs. 41 and 42 are satisfied:

$$\frac{1+k}{2}\delta_s^H + \frac{1-k}{2}\delta_s'^H = H \tag{41}$$

$$\frac{k}{2}\delta_s^L + \frac{2-k}{2}\delta_s'^L = L \tag{42}$$

The constants H and L are selected based on the primary and secondary misvaluation levels of the buyers, and set in such a way that a seller must initially update the misvaluation levels. If we assume that $\frac{1+k}{2}\delta_s^L + \frac{1-k}{2}\delta_s'^L < H$ and $\frac{k}{2}\delta_s^H + \frac{2-k}{2}\delta_s'^H > L$, then the seller will need to update for a second time upon meeting another buyer. Now let us consider the situation where the seller increases, whenever necessary, the first-order misvaluation level by adding the constant $\Delta_H - (1 - k) \times \epsilon$ and the second-order misvaluation level by adding $\Delta_H + (1 + k) \times \epsilon$. The sellers prefer to update their beliefs about the value distribution of the buyers than their own valuation. The different weights are due to the fact that the condition given in Eq. 41 must be satisfied after the updating process. On the other hand, the seller decreases, whenever necessary, the first-order misvaluation level by subtracting the constant $\Delta_L - (2 - k)/2 \times \epsilon$ and the second-order misvaluation level by subtracting $\Delta_L + k/2 \times \epsilon$. The different weights are due to the fact that the condition given in Eq. 42 must be satisfied after the updating process. When this eventuality occurs (meeting a buyer with the highest possible value and then a buyer with the lowest possible value), the difference between the second-order and first-order misvaluation levels will increase by ϵ in one cycle. As Eqs. 41 and 42 must be satisfied alternately, the sellers will adjust the misvaluation levels infinitely many times, wherein the second-order misvaluation level will tend to (positive) infinity and the first-order level to minus infinity.

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